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ANCHORING OR LOSS AVERSION?

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Anchoring or Loss Aversion? Empirical Evidence from Art Auctions

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Abstract

We find evidence for the behavioral biases of anchoring and loss aversion. We find that anchoring is more important for items that are resold quickly, and we find that the effect of loss aversion increases with the time that a painting is held. The evidence in favor of anchoring and loss aversion with this large dataset validates previous results and adds to the empirical evidence a finding of increasing loss aversion with the length a painting is held. We do not find evidence that investors can take advantage of these behavioral biases.

Keywords: anchoring, loss aversion, endowment effect, art auctions

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1 Introduction

The seminal work of Daniel Kahneman and Amos Tversky has shown repeatedly that individuals use heuristics, such as a anchoring on a previous sale price, when solving difficult problems and that these heuristics lead to biased judgement. Their work on loss aversion has demonstrated that individuals dislike monetary losses more than they enjoy monetary gains. Because works of art are unique and difficult to value and because there is a large amount of empirical data available from over a hundred years of art auctions, art prices are a good medium with which to study the behavioral biases of anchoring and loss aversion. As the same work of art can be sold repeatedly over many decades — or even centuries — and since auction sales are publicly recorded, it is also possible to shed light on whether or not investors could have taken advantage of these behavioral biases within our auction dataset.

Anchoring was first proposed by Amos Tversky and Daniel Kahneman (1974). Subjects were given a number — determined by the spin of a wheel — between one and 100. They were then subsequently asked the number of African countries in the United Nations. The subjects showed a bias towards the original number they were given. In general, anchoring refers to an irrelevant message having an effect on the outcome. Alan Beggs and Kathryn Graddy (2009) show that it matters whether a painting had been previously sold in a 'hot' market vs. a 'cold' market using a limited dataset of repeat sales.

Loss aversion (Kahneman and Tversky (1979)) is related to anchoring, but with asymmetric effects depending upon whether the price has subsequently increased or decreased. Rather than expected utility where gains and loses are perceived with the same magnitude, losses weigh more heavily in the mind of individuals. In a classic demonstration of loss aversion, David Genesove and Christopher Mayer (2001) show that home owners set higher asking prices for condominiums for which they have suffered nominal losses.

The time lapsed between two sales of an identical item can have an effect on the salience of the previous price as an anchor and can also have an effect on loss aversion. The tendancy to place a larger value on an item in one's possession is called the endowment effect, which was introduced by Richard Thaler (1980) and has been studied extensively since. Michael Strahilevitz and George Lowenstein (1998) find that for attractive items loss aversion tends to be greater the longer the good is held.

This research extends previous work on anchoring and loss aversion in three ways. First, it replicates previous results on anchoring and loss aversion using a much larger dataset of repeat art sales extending over a long period of time. This empirical study adds to a relatively small body of work that tests for behavioral biases using actual data rather than experimental settings. Second, when using the previous sale price as an anchoring point, the research shows that the degree of anchoring and loss aversion may depend upon the time between repeat sales, which is correlated with both the salience of the previous sale and the holding period. Finally, we examine whether the behavioral biases of anchoring and loss aversion can be used to predict future returns.

When testing for anchoring or loss aversion, it is critical to separate out anchoring (where an 'irrelevant' message has an effect on the outcome) from rational learning (where past prices are important because they represent unobservable quality effects). The identification strategy we use in this paper is similar to the strategy used in Genesove and Mayer (2001) and again later in Beggs and Graddy (2009). We can identify anchoring from other effects because the demand for art, which is captured by the average overall price index, changes over time, whereas the unobservable component of quality is assumed to remain constant between auctions. This allows us to control for unobserved quality characteristics. As long as something drastic has not happened between sales — such as a painting has been deemed a fake, which is a very rare occurrence—the assumption of constant quality is a realistic one.

This paper proceeds as follows. Section two discusses anchoring, loss aversion, and these behavioral biases over time. The model and the tests for behavioral anomalies are described in the third section. In section four we present the data and summary statistics. Section five introduces the hedonic model for estimating predicted prices. In section six we discuss our empirical findings; section seven discusses excess returns and section eight concludes our analysis.

2 Art Auctions and Behavioral Biases

2.1 Art Auctions

Art can be sold either through a dealer or through an auction house. The major auction houses are Christie's and Sotheby's, and the auction is conducted in the English, ascending auction format. Auction houses generally charge a buyer's premium of between 12.5 percent and 25 percent of the painting. The seller's premium is unknown and subject to negotiation.

Before the auction starts, the auctioneer will determine a pre-sale low and high estimate for the work of art and will publish the details of the work of art both online and in a catalogue. The pre-sale estimates are determined in conjunction with the seller; the seller also sets a secret reserve price that is equal to or below the pre-sale low estimate. If the bidding does not reach the secret reserve price, then a painting is said to be 'bought in'. The painting is not actually 'bought in' by the auction house, but simply goes unsold and is returned to the seller. The seller can decide to list the item again at auction, take it to a dealer, or take it off the market. About 30 percent of paintings on average are 'bought-in.'

2.2 Anchoring

The anchoring heuristic was first introduced by Tversky and Kahneman (1974). With few exceptions, most instances of anchoring have been introduced in laboratory or experimental settings. Some well known laboratory experiments in the field of economics include Dan Ariely, George Lowenstein and Drazen Prelec (2003), Drew Fudenberg, David Levine and Zacharias Maniadis (2012), and Abhijit Banerji and Neha Gupta (2013). Maniadis, Fabio Tufano, and John List (2014) demonstrate the limitations of experimental results on anchoring.

Gregory B. Northcraft and Margaret A. Neale (1987) were one of the first authors to use actual market data to demonstrate anchoring by investigating the effect of manipulating the alleged list price on valuations of properties by estate agents. This study, however, was not performed in a true market context as it involves valuations, not prices. K. N. (Raj) Rajendran and Gerard J. Tellis (1994) and Eric A. Greenleaf (1995) examine the importance of past prices when consumers repeatedly purchase the same commodity. Beggs and Graddy (2009) use actual market data on art auctions to demonstrate anchoring, and subsequently, anchoring has been studied in commercial real estate by Sheharyar Bokhari and David Geltner (2011). More recently, Dougal et al. (2012) have shown that anchoring influences the cost of capital.

The principle of anchoring on a previous price can be applied to sellers, buyers, or auctioneers. For example, it is possible that buyers are directly influenced by previous sale prices or that auctioneers' pre-sale estimates anchor on previous sale prices, and the buyers (and sellers with their undisclosed reserves), are influenced by auctioneers' expert opinions. In many ways, art and real estate are natural goods in which to study anchoring on previous prices. Both types of markets involve goods that are unique and difficult to value. In these types of goods, it is natural that individuals use a heuristic when making decisions on price. There are, of course, many different prices on which participants in auctions can focus. These include estimates, reserve prices (Stephanie Rosenkranz and Patrick Schmitz (2007)) and buy prices (Nicholas Shunda (2009)), as well as endogenously determined reference prices (Andreas Lange and Anmol Ratan (2010), and Husnain Ahmad (2013).

2.3 Loss Aversion

In a series of papers on prospect theory, Kahneman and Tversky (1979) show that the outcome of risky prospects are evaluated by a value function with three characteristics: first, gains and losses are dependent on a reference point; second, the value function is steeper for losses than for equivalently sized gains (loss aversion); and third, diminishing sensitivity to gains and losses. The role of a 'loss' was first studied in a number of experimental papers, providing strong evidence that people prefer to avoid realizing losses. The empirical evidence resulting from observing the outcomes of pairs of concurrent decisions is that the magnitude of the impact of losses over gains results in losses being weighted just over twice as heavily as gains.

Indeed loss aversion has been studied much more widely outside an experimental setting than anchoring. In addition to Genesove and Mayer (2001) and Bokhari and Geltner (2011), loss aversion has been studied extensively in the finance literature including papers by Schlomo Benartzi and Thaler (1995) and Nicholas Barberis, Ming Huang, and Tano Santos (2001) on the stock market, Mao-Wei Hung and Jr-Yan Wang (2011) on interest rates, and Kenneth Froot et. al. (2011) in currency markets. Piet Eichholz and Thies Lindenthal (2012) look at loss aversion in real-estate markets over time and across generations.

Brunno Jullien and Bernard Salanie (2000) look at the effect of loss aversion on race track betting. Our research is one of few attempts to measure differences in effects due to loss aversion and anchoring.

Loss aversion can result in a reluctance to realize a loss (Hersh Shefrin and Meir Statman, 1985). As loss aversion encourages owners to increase their secret reserve price, some items will not sell immediately at auction, resulting in these owners holding these items for a longer period of time.

At first glance, the principle of 'loss aversion' is an idea that applies to sellers; sellers are the individuals who are averse to realizing actual losses. However, much of the recent theoretical literature shows that buyers can be considered loss-averse relative to some endogenous reference point. In this case, the endogenous reference point could be influenced by the previous sale price. This literature draws on the expectations-based model of Botond Koszegi and Mathew Rabin (2006). Applications of endogenously determined reference points in an auction context include Lange and Ratan (2009), Shunda (2009), and Ahmad (2013). Furthermore, in art auctions sellers almost always set a secret reserve price. This secret reserve price can end up raising the auction price if the bidder with the second highest reservation value has a valuation below the secret reserve, and the highest bidder's valuation is above the secret reserve. In cases where the highest bidder's reservation value is below the secret reserve, the item will go unsold.

2.4 Loss Aversion and Anchoring Over Time

The tendency to place a larger value on an item when it is in one's possession is called the "endowment effect", which was introduced by Thaler (1980). Strahilevitz and Lowenstein (1998) take this one step further by studying the effect of ownership history on the valuation of objects. They find that one's valuation increases with the duration of ownership, in contrast to the "instant endowment" effect as labelled by Kahneman et. al. (1990). Ownership makes an individual increasingly averse to losses. This idea is reaffirmed and extended by Brenner et. al. (2007) in a paper that shows that this effect is stronger for attractive items.

3 Testing for Anchoring and Loss Aversion

Testing for anchoring and loss aversion consists of explaining the natural log of the hammer price or the natural log of the estimated price by a predicted price, an anchoring effect, a loss aversion effect and an unobservable quality effect, as used by Beggs and Graddy (2005).[1]

The equation which we estimate to test for anchoring and loss aversion for each painting sold at time t has the following form in log prices:

$$p_{it} = a_0 + a_1\pi_{it} + a_2(p_{it-1} - \pi_{it}) + a_3(p_{it-1} - \pi_{it-1}) + a_4\Phi(\phi, \tau) + \eta_{it} \qquad (1)$$

The first term of equation 1 above, a_0, is a constant. The second term, the predicted price, π_{it}, is constructed from a hedonic price model, $\pi_{it} = \mathbf{X_i B} + \delta_t$, where $\mathbf{X_i}$ represents characteristics of work i, $\mathbf{B}$ is a vector of coefficients on these characteristics, and δ_t is a time effect.

The third term, $a_2(p_{it-1} - \pi_{it})$, captures the anchoring effect. If there is anchoring, then the final price will be adjusted by a proportion of the difference between the predicted price and the previous price. The adjustment will be identical whether the predicted price is above or below the previous price.

The fourth term, $a_3(p_{it-1} - \pi_{it-1})$, in equation 1 controls for unobservables.

[1] Genesove and Mayer (2001) first developed a version of this method but did not include an anchoring effect. Beggs and Graddy (2009) included an anchoring effect, but not a loss aversion effect.

Because the econometrician cannot observe the actual predicted price, there is an unobservable effect in this term that is correlated with the previous sale price of the painting. The identifying assumption is that the characteristics stay the same over time. In this case, the unobservable effect will be equal to the difference between last period's price and last period's prediction.

The fifth term in the equation, $a_4\Phi(\phi, \tau)$, is the loss aversion effect, which captures the differential effect of a loss relative to a gain. The loss aversion term is the absolute value of the expected loss, if there is a loss; otherwise the term is zero. Because there is an unobservable quality effect in this nonlinear loss term, the quality effect cannot be easily factored out as it was in the anchoring term. Thus, a nonlinear estimate is required. As explained in more detail in the appendix, ϕ and τ are the parameters that define the distribution of the sellers' ex-ante expected loss. A significant and positive coefficient on this term indicates loss aversion. If the loss aversion effect is zero, then there is symmetry between gains and losses, and the term can be dropped; this is then equivalent to the simple model estimated in Beggs and Graddy (2009).

We allow for heteroskedasticity of the error term, η_{it}. For more information on the specification, please see the appendix.

In addition to estimating the full nonlinear model below, we also estimate a linear version of the model where the unobservable effects in the anchoring term are ignored. Genesove and Mayer (2001) demonstrate theoretically that the linear coefficients on the anchoring effect in the linear model provide a lower bound for the anchoring effects in the nonlinear regression.

We estimate equation 1 with two different dependent variables, log price and log estimate.

4 Data and Summary Statistics

The data come from a database put together by Jianping Mei, Mike Moses and Rachel Pownall, and are used to construct the Mei Moses Fine Art Index®. Parts of the data have been used in various academic papers, including Mei and Moses (2002, 2005) and De Silva, Pownall and Wolk (2012).

Each observation in the data represents a purchase and a sale of the same painting, so each observation provides information on two prices, a purchase price and a sale price. Some paintings have more than one observation, implying that there were more than two prices of that painting recorded. In this case, a sale in one observation appears as the purchase in another observation.

Table 1 provides summary statistics on the purchase price, the sale price, the purchase and sale high estimate and low estimate, and the months since the last sale. All prices are in US dollars. (Note: all price pairs are converted to end of year US prices). Clearly the auction sales prices indicate that the paintings sold are from the high end of the market. Sotheby's and Christies, the two major art auction houses for the New York and London art markets, represent a large fraction of the data sample. The average sales price is \$751,172 and the average purchase price is \$289,406, with an average holding period of about 16 years. Note that a painting may have changed hands through a private sale or may have been bequeathed to a new owner between the first and second recorded auction sales. It is also possible that a painting came to auction and did not sell between the two observed sales. To the extent that anchoring and loss aversion are driven by the seller's knowledge of the previous price, this could potentially provide a downward bias to the data, as the regressors would be conditioning on the "wrong" previous price. However, it is likely that any downward bias is small as the large majority of important sales of art (unlike violins!) take place at auction and the buy-in rate (the rate at which paintings go unsold because they do not meet their reserve price) is low in a sample of repeat sales that contain relatively important pieces. In addition, the buyer and the auctioneer may be using the published and readily accessible auction price as the reference, even in the presence of a previous sale. We therefore do not take account of this potential downward bias in our regressions or interpretations below.

In addition to information on the sale price, date sold, and the auction house's low and high estimates, the dataset includes information about the artist such as birth year, death year and country. It also includes descriptive information about the painting, such as its dimensions, shape, medium, whether it was signed and dated and the date it was painted.

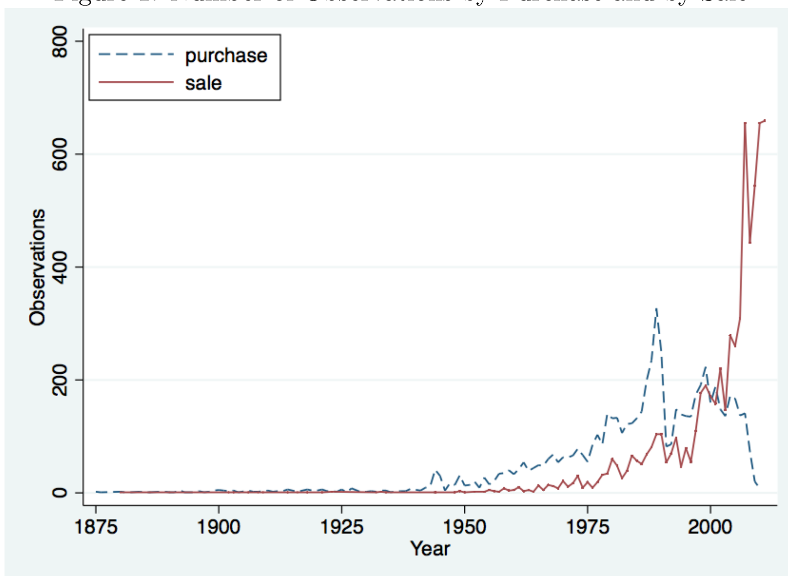
Table 1: Summary statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
Purchase price	289406	1172048	4	29150000	6411
Sale price	751172	2878625	23	80451176	6411
Sale high estimate	707047	2423097	735	59080000	5899
Sale low estimate	513670	1755471	441	44310000	5899
Purchase high estimate	386364	1164394	700	20000000	3306
Purchase low estimate	281085	854031	440	16000000	3318
Sale year	2001	12	1880	2011	6411
Purchase year	1985	19	1875	2010	6411
Holding period	16.4	13.8	0.6	123.3	6411

The entire dataset starts in 1875 and goes through December of 2011.[2] However, the purchases and sales are not spread evenly over the time period, with purchases taking place, by definition, earlier than sales. Figure 1 below documents the purchases and sales over the time period of the data.

[2]We are using nominal prices as the previous price provides a strong focal point on which to anchor. Furthermore, when Beggs and Graddy (2009) deflated by CPI, there was no change in the estimate because changes in art prices swamp changes in CPI. For similar reasons, pricing anomalies, such as the Christie's and Sotheby's commission-fixing scandal that took place between 1993 and 2000 (see Ashenfelter and Graddy (2005)) would be too small to affect the overall results.

Figure 1: Number of Observations by Purchase and by Sale



5 Hedonic Regressions

In order to conduct the empirical tests set out in section 3, it is first necessary to construct a predicted price for the current sale and a predicted price for the purchase. One method to construct a predicted price is by hedonic regression, in which the log price is regressed on various painting characteristics as well as year fixed effects. The characteristics that are used in the hedonic regressions are length, width, whether the painting is signed, whether the painting is dated, and fixed effects for the medium, the artist, and the year.

The regression results from the hedonic regression are presented in Table 2. The coefficients from this regression are used to construct a predicted price for each painting.

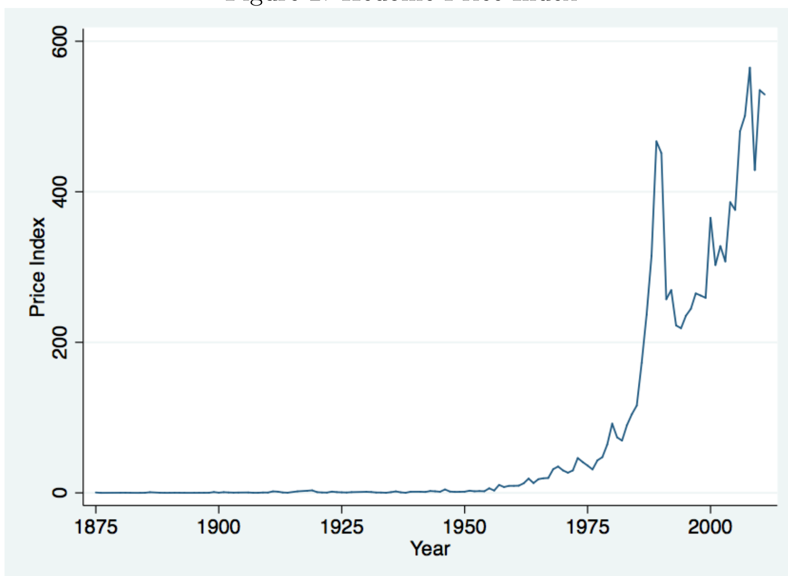
Table 2: Hedonic Regression

Dependent Variable: ln (Sale Price)	
Painting is Dated	.196
	(.023)***
Log of Width (inches)	.628
	(.031)***
Log of Height (inches)	.799
	(.031)***
Painting is Signed	.149
	(.037)***
Medium Fixed Effects	yes
Artist Fixed Effects	yes
Year Fixed Effects	yes
Obs.	12536
R^2	.793

The value of a painting increases as both the height and the width increases, and signed and dated paintings both have significant positive effects. The artist, medium and time fixed effects are each jointly statistically significant at the .001 level. Note that the number of observations is not quite twice as high as the number of observations when each painting is grouped as a purchase and a sale, as a repeat sale. This discrepancy is because some paintings are sold more than twice. Below is a graph of the price index, calculated as the exponential of the coefficients from the year fixed effects in the above hedonic regression, from 1875 to the present.[3]

[3]An interesting check would be to test whether individuals were anchoring on real prices or nominal prices by deflating prices by the art index (rather than CPI). However, it is changes in the art index that identify the anchoring and loss aversion effects from the unobservable

Figure 2: Hedonic Price Index



Although the general trend has been up, the index indicates ample opportunity for both increases and decreases in price. The pattern is similar to other estimates of art price indices, such as Mei and Moses (2005) and Ronneboog and Spaenjers (2013). The bubble in the late 1980s is most often explained by the increase in wealth driven by asset prices. The subsequent fall in wealth and the withdrawal of the Japanese from the art market after the Japanese stock market crash is often cited for this decline. Goetzmann and Renneboog and Spaenjers (2011) provide a discussion and analysis of the relationship of art to income and wealth.

6 Empirical Results

In this section, we estimate equation 1 above and attempt to separate out anchoring from loss aversion using both the log sale price and the log of the low

term.

estimate as dependent variables. In Table 3 we present the regression results for the full sample, and in Table 4, we present results for the sample split at five years.

Note that this model, as in Genesove and Meyer (2001), Beggs and Graddy (2005 and 2009), Bokhari and Geltner (2011), and other work relies on the assumption that the correct model for price in the absense of anchoring and loss aversion is a prediction of price, adjusted by the difference between the prediction the previous time the work was sold and the previous price:

$$p_{it} = b_1 \pi_{it} + b_2 (p_{it-1} - \pi_{it-1}) \tag{2}$$

We begin with this specification in columns 1 and 4 of Table 3 below.

6.1 Full Sample

As a preface to our main regression, we can see from column 1 that when we estimate equation 2, the coefficients on the predicted price are statistically significantly different from one, though are very close to one in value. These coefficients are reassurring in the absence of other regressors as they indicate that modeling price as the predicted price plus an adjustment is at least consistent with the data.[4]

In columns 2 and 5 we estimate a linear version of the regression where we ignore the unobservables in the loss aversion term, and in columns 3 and 6, we present our non-linear results. Our nonlinear estimation technique is described in detail in the Appendix. All standard errors reported are robust standard errors, and all variables are in natural logarithms.

[4]These equations are estimated without a constant. When a constant is included, the coefficient on the constant is -0.71 in column 1 and -1.29 in column 4; both coefficients are significantly different from zero. However, the coefficients on predicted price remain close to one in value (1.06 in column 1 and 1.08 in column 4). A better model would include a constant.

In this specification, the coefficient on the predicted price is close to one in magnitude, as it should be. Furthermore, the coefficient on unobservable quality is large and highly significant; if, for example, the price was 100 percent greater than the predicted price during the previous sale, then this in itself will raise the price for the current sale by a little over 50 percent. Finally, the longer the holding period for a work of art, the higher the price: each ten year increase in holding period increases the price by around 6 percent.

The regression results indicate that both anchoring effects and loss aversion effects are present, though the anchoring coefficient is only statistically significant in the nonlinear results. An interpretation of the anchoring and loss aversion results in the nonlinear regressions is that a 10 percent unrealized or expected loss (that is, a positive difference between the previous price and the current predicted price), results in a 6.2% increase in price relative to what the sale price would have been in the absence of anchoring and loss aversion. Of this difference, 0.9 percent is due to anchoring, and 5.3 percent of this difference is due to loss aversion. A 10 percent expected gain (that is, a negative difference between the previous price and the predicted price) results in a 0.9 percent decrease in price relative to what the sale price would have been in the absence of anchoring.

The larger loss aversion coefficient in the nonlinear regression is consistent with the Genesove and Mayer (2001) prediction.

Here it is not possible to tell whether the sale price adjustments result because of anchoring on the part of the buyer, the seller, or the 'expert' auctioneer. Generally the pre-sales estimate is set by the seller in consultation with the auction house, which gives an indication of the extent of the anchoring on the sellers behalf. However, it is instructive to see if results using the pre-sale estimate are consistent with the above results.

Table 3: Linear and Nonlinear Regression Results

	Sale Price			Low Estimate		
	Linear	Linear	Nonlinear	Linear	Linear	Nonlinear
	(1)	(2)	(3)	(4)	(5)	(6)
Predicted Price	1.005***	1.053***	1.058***	0.974***	1.070***	1.078***
	(0.001)	(0.007)	(0.006)	(0.001)	(0.007)	(0.006)
Anchoring		0.011	0.088***		-0.009	0.065***
		(0.019)	(0.013)		(0.019)	(0.013)
Loss Aversion		0.457***	0.533***		0.444***	0.600***
		(0.037)	(0.044)		(0.040)	(0.095)
Unobservable Quality	0.664***	0.548***	0.518***[5]	0.688***	0.599***	0.568***
	(0.015)	(0.024)		(0.016)	(0.025)	(0.026)
Holding Period		0.006**	0.008***		0.002	0.004***
		(0.002)	(0.001)		(0.002)	(0.001)
Constant		-0.727***	-0.733***		-1.268***	-1.308***
		(0.077)	(0.065)		(0.087)	(0.070)
$1-\phi$			0.432***[6]			0.392***[7]
$\ln(\tau)$			-2.519			-2.519
Obs.	6411	6411	6411	5899	5899	5899
R^2	*	0.850	0.853	*	0.845	0.879

We find that the results from regressions using the log of the pre-sale low estimate as the dependent variable are very similar to the regressions when

[5] 99 percent confidence interval based on bootstrapped percentiles: $[0.5063, 0.5871]$
[6] 99 percent confidence interval based on bootstrapped percentiles: $[0.3819, 0.4422]$
[7] 99 percent confidence interval based on bootstrapped percentiles: $[0.3216, 0.4146]$

price is used as a dependent variable. The effect of loss aversion on the low pre-sale estimate is not significantly different than when the sale price is used, though the point estimate is slightly larger. As the pre-sale low estimate is set in conjunction with the seller's secret reserve, this would be expected.

One initial interpretation of these regressions is that the pre-sale estimates are 'correct' (for items that are sold), and that auction house experts do a good job in predicting current prices. That is, the estimates accurately reflect the determinants of price. The hedonic pricing function is a good indicator of the pre-sale estimate and the coefficients are similar in magnitude to those in columns 1 and 2.

The coefficients of interest from the above table — the coefficients on anchoring and loss aversion — contrast with previous results of Beggs and Graddy (2005, 2009) who only find symmetric anchoring effects. In their study, the coefficient on loss aversion is not significantly different from zero. While we find anchoring effects in this paper in the nonlinear specification, we also find significant asymmetric effects that are attributable to loss aversion.

There are two primary differences between this dataset and the dataset used in Beggs and Graddy (2005, 2009). First, this dataset is much, much larger, with 6411 repeat sales observations as opposed to only 76 observations used in the anchoring regressions for the Impressionist Art dataset in Beggs and Graddy (2005, 2009). The fact that Beggs and Graddy (2005, 2009) do not find significant loss aversion effects could very well be due to the very small sample size. A finding of anchoring in their study is notable given the small size of the sample.

6.2 Split Sample Results

The second primary difference between the two datasets is the holding period. In the dataset used in this paper, the average time between sales is about 16 years. In Beggs and Graddy (2005, 2009), the average time between sales is just over 3 years. When Beggs and Graddy (2009) restrict the time sample to sales that took place within their 3 1/2 year period, the anchoring effects become stronger, especially in the very small (22 observations) Contemporary Art sample.

In Tversky and Kahneman (1982), salience is an important part of biases in judgement. One would expect an anchor to be more salient if a work was sold relatively recently rather than a long time ago. However, the endowment effect may become stronger if a painting is held for a long period of time, as noted by Strahilevitz and Lowenstein (1998).

Running the regression in equation 1 over the two subsamples, we observe the anchoring and loss aversion behavior for short and long holding periods. In Table 4, we present these two subsamples. When paintings are resold relatively quickly, within a 5 year period, we find the coefficient for anchoring to be significantly larger for shorter holding periods in both the linear and nonlinear regressions when the natural log of the sale price is used as the dependent variable. This result is consistent with salience: for the short term period, salience of the purchase price is likely to lead to the significantly larger coefficient. Interestingly, when the natural log of the low estimate is used as the dependent variable, the anchoring effects are not as large for the shorter time period, though the point estimates are still larger than the point estimates for the longer time period.

For loss aversion, for the linear regression we find that for both dependent variables, the natural log of the sale price and the natural log of the low estimate,

Table 4: Linear and Nonlinear Regression Results with One Break at 5 Years

	Sale Price				Low Estimate			
	Linear	Nonlinear	Linear	Nonlinear	Linear	Nonlinear	Linear	Nonlinear
	< 5	< 5	≥ 5	≥ 5	< 5	< 5	≥ 5	≥ 5
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Predicted Price	1.021***	1.036	1.059***	1.065	1.043***	1.044	1.075***	1.087
	(0.012)	0.009	(0.008)	0.007	(0.011)	0.009	(0.009)	0.007
Anchoring	0.153*	0.189***	0.010	0.085***	0.071	0.131***	-0.004	0.065***
	(0.068)	(0.059)	(0.020)	(0.014)	(0.071)	(0.064)	(0.021)	(0.015)
Loss Aversion	0.233***	0.485***[8]	0.458***	0.392***[9]	0.200***	0.340[10]	0.449***	0.475**[11]
	(0.064)		(0.045)		(0.057)		(0.047)	
Unobservable Quality	0.587***	0.611***[12]	0.532***	0.510***[13]	0.712***	0.708***[14]	0.579***	0.574***[15]
	(0.065)		(0.026)		(0.072)		(0.027)	
Holding Period	0.021	0.004	0.006**	0.008***	-0.021	-0.021***	0.003	0.005***
	(0.015)	(0.012)	(0.002)	(0.001)	(0.012)	(0.012)	(0.002)	(0.001)
Constant	-0.300*	-0.402***	-0.810***	-0.827***	-0.756***	-0.734***	-1.341***	-1.427***
	(0.136)	(0.112)	(0.091)	(0.078)	(0.140)	(0.120)	(0.102)	(0.082)
$1-\phi$		0.246***[16]		0.442***[17]		0.186[18]		0.387***[19]
$\ln(\tau)$		-2.519		-2.519		-2.519		-2.519
Obs.	1087	1087	5324	5324	946	946	4953	4953
R^2	0.928	0.931	0.833	0.836	0.935	0.953	0.831	0.861

[8] 99 percent confidence interval based on bootstrapped percentiles: $[0.3841, 0.5405]$
[9] 99 percent confidence interval based on bootstrapped percentiles: $[0.3630, 0.4025]$
[10] 90 percent confidence interval based on bootstrapped percentiles: $[-0.0494, 0.7972]$
[11] 95 percent confidence interval based on bootstrapped percentiles: $[0.0592, 0.5548]$
[12] 99 percent confidence interval based on bootstrapped percentiles: $[0.4294, 0.7591]$
[13] 99 percent confidence interval based on bootstrapped percentiles: $[0.4467, 0.5979]$
[14] 99 percent confidence interval based on bootstrapped percentiles: $[0.0068, 0.9578]$
[15] 99 percent confidence interval based on bootstrapped percentiles: $[0.1691, 0.8373]$
[16] 99 percent confidence interval based on bootstrapped percentiles: $[0.1407, 0.3618]$
[17] 99 percent confidence interval based on bootstrapped percentiles: $[0.3668, 0.4975]$
[18] 99 percent confidence interval based on bootstrapped percentiles: $[0, 0.9678]$
[19] 99 percent confidence interval based on bootstrapped percentiles: $[0.1106, 0.7638]$

that loss aversion appears to have a statistically larger effect for the longer periods than for the shorter periods. However, we find no significant difference in coefficients for the nonlinear results, though for the low estimate, the point estimate on loss aversion only becomes statistically significant for the longer holding period. The strength of the low estimate results may be due to the fact the the low estimate is set in conjunction with the seller and must be at or above the seller's secret reserve price. The above results provide convincing evidence that the anchoring effect is stronger for shorter holding periods, and some evidence that the loss aversion effect is stronger for longer holding periods.

7 Excess Returns

In the previous section we found that the sale price increases when the estimate for a work is difficult to predict by hedonic methods, but that pre-sale estimates can increase even more for these works, resulting in unsold paintings. Furthermore, we find that time between sales increases the sale price and that paintings in which the seller has a loss also tends to increase prices through behavioral effects. Does this mean that we can predict excess returns? We test this notion by putting together yet another unique dataset of items that have been sold three times or more. The dataset is summarised in Table 5 below.

Table 5: Summary statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
Sale price 0	207364	898942	21	10780000	583
Sale price 1	553771	1866588	23	22552500	583
Sale price 2	1199527	4226424	75	68962496	583
Sale year 0	1971	24	1875	2006	582
Sale year 1	1986	18	1880	2009	583
Sale year 2	2000	12	1886	2011	583
Holding period 1	15	15	1	90	582
Holding period 2	14	14	-14	97	583

As this dataset contains is a subset of our first dataset, we first re-estimate equation 1. This replication is presented in column 1 of Table 6 below. The results are very similar to our previous results, with the exception that the point estimates of the anchoring effect and the holding period are smaller and not significant in this smaller sample.

Secondly, we test whether any of these components — the unobservable component, the anchoring and loss component, or the holding period — has an effect on future excess returns in an out-of-sample regression. Specifically our regression is as follows, where the prices at the three sales are denoted p_0, p_1, and p_2 respectively, and years between the first two sales and second two sales are denoted T_1 and T_2.

$$\frac{p_2 - p_1}{T_2} - \frac{p_2^m - p_1^m}{T_2} = \beta_0 + \beta_1(\pi_1) + \beta_2(p_0 - \pi_0) + \beta_3(p_0 - \pi_1) + \beta_4(p_0 - \pi_1)^+ + \beta_5(T_1) + \epsilon_\beta \quad (3)$$

The results are presented in column 2 of Table 6. Despite the fact that loss aversion significantly increases price as shown in column 1, this increase in price does not translate into a decrease in returns.

However, if a painting trades for much higher than its predicted price in the second sale, this is likely to significantly decrease excess returns generated by the third sale. Likewise, if a painting trades for much lower than its predicted price in the second sale, this is likely to significantly increase excess returns generated by the third sale. Thus, we find strong evidence that over or underpayment, as defined by the difference between the actual price and the predicted price, provides a reliable predictor of future returns.

Out of curiosity, we replicated the regression in column 2 using the low pre-

Table 6: Excess Returns

	Saleprice	Excess
	(1)	(2)
Predicted Price 1	1.000	.001
	(.016)***	(.003)
Anchoring	-.023	.008
	(.042)	(.006)
Loss Aversion	.529	.003
	(.090)***	(.014)
Unobservable Quality	.434	-.025
	(.050)***	(.009)***
Holding Period 1	.006	.0007
	(.004)	(.0006)
Const.	-.182	-.016
	(.178)	(.035)
Obs.	496	496
R^2	.913	.017

sale estimate for the predicted price, rather than our prediction. We did not find that the difference between the final price and the low pre-sale estimate was a predictor of future excess returns.

8 Conclusion

In this paper, we find evidence for the behavioral biases of anchoring and loss aversion. We find that anchoring is more important for items that are resold quickly, and we find that the effect of loss aversion increases with the time that a painting is held. The evidence in favor of anchoring with this large dataset validates previous results from Beggs and Graddy (2009) on anchoring and from Genesove and Mayer (2001) on loss aversion and adds to the empirical evidence a finding of increasing loss aversion with the length a painting is held.

A contribution of this paper is that not only can we identify behavioral biases, but we also have the data to test whether investors can take advantage of these behavioral biases. We do not find any evidence that this is the case. It does not appear to be the case that excess returns be earned by understanding these behavioral biases.

However, we do find that paintings whose previous prices have deviated positively from a previous predicted price have a higher probably of earning lower returns. Equivalently, we find that paintings whose prices have deviated negatively from a previous predicted price earn higher returns.

Appendix: Motivation of Estimating Equation and Explanation of Nonlinear Least Squares (NLS) Estimation

The goal of this appendix is to motivate equation 1, the estimating equation. We do this first by explaining the theory behind the estimation, which is largely adapted from Beggs and Graddy (2005). We then explain the details of the nonlinear lease squares estimation, starting with the grid search and ending with the inference.

The Estimating Equation

We start with the assumption that the auctioneer believes the true model for value of each painting i is given by:

$$p_{it} = \pi_{it} + u_i + \epsilon_t$$

Both u_i (unobserved value) and ϵ_t (shocks to that particular period) are i.i.d. normal, and $\pi_{it} = \mathbf{X_i B} + \delta_t$,

The auctioneer observes p_{it-1}, π_{it-1}, and a signal w_i—representing the knowledge of the auctioneer. w_i is jointly normally distributed with u_i.

In a world with perfect information, $w_i = u_i$. But in a world with imperfect information, we would expect the auctioneer's estimate to satisfy the following equation:

$$Est_{it} = \pi_{it} + v_i$$

where

$$v_i = E(u_i | \pi_{it-1}, p_{it-1}, w_i)$$

The econometrician does not observe the auctioneer's estimate of quality: v_i. Since v_i depends on the painting's previous price and price estimate, this gives a conditional estimate of the unobserved quality:

$$E(v_i | \pi_{it-1}, p_{it-1}) = E(u_i | \pi_{it-1}, p_{it-1})$$

Furthermore, from joint normality of w_i and u_i, we have:

$$E(u_i | \pi_{it-1}, p_{it-1}) = \phi(p_{it-1} - \pi_{it-1})$$

where ϕ is a constant. If the variance of u_i and ϵ_t are σ_u^2 and σ_ϵ^2 respectively, then $\phi = \sigma_u^2/(\sigma_u^2 + \sigma_\epsilon^2)$. The closer ϕ is to 1, the larger the uncertainty in unobserved quality in comparison to idiosyncratic shocks.

We can then write:

$$Est_{it} = \pi_{it} + \phi(p_{it-1} - \pi_{it-1}) + \omega_{it}$$

where ω_{it} is orthogonal to the other terms.

This equation is then updated to account for both anchoring and loss aversion. Similar techniques apply for including a term to account for anchoring, since it is linear. However, things become more complicated when accounting for loss aversion, because it is nonlinear.

The ideal equation to estimate is the following:

$$Est_{it} = \pi_{it} + v_i + \lambda(p_{it-1} - v_i - \pi_{it}) + v(p_{it-1} - v_i - \pi_{it})^+$$

Here the third term accounts for anchoring and the last term captures loss aversion. Without the loss aversion term, the equation can still be estimated using OLS by taking expectations of the equation conditional on observed variables and using properties of conditional expectations to rewrite the equation with v_i in the error term.

However, because the loss aversion term is the positive part of a difference of observed and unobserved (v_i) variables, we cannot apply the same technique. Assuming the auctioneer observes quality perfectly, implying that $u_i = v_i$, we get:

$$(p_{it-1} - \pi_{it} - v_i)^+ = (\pi_{it-1} - \pi_{it} + \epsilon_{t-1})^+$$

Conditional on p_{it-1} and π_{it-1}, ϵ_{t-1} is Normal with mean $(1 - \phi)(p_{it-1} - \pi_{it-1})$.

The expectation of $(p_{it-1} - \pi_{it} - v_i)^+$ conditional on the observable variables is then equal to the expectation of U^+ where U has a normal distribution with mean

$$\pi_{it-1} - \pi_{it} + (1 - \phi)(p_{it-1} - \pi_{it-1})$$

and some variance τ^2. Under the assumptions above $\tau = \sigma_\epsilon^2\phi$. We denote the expectation of U^+ with $z(\phi, \tau)$. We add a constant to the equation to be estimated and rewrite the estimating equation as:

$$Est_{it} = a_0 + a_1\pi_{it} + a_2(p_{it-1} - \pi_{it}) + a_3(p_{it-1} - \pi_{it-1}) + a_4\Phi(\phi, \tau) + \eta'_{it}$$

where η'_{it} is orthogonal to the other regressors and has zero conditional expected value. The above theory implies two restrictions on the coefficients: that ϕ lies between 0 and 1 and that the coefficient on the unobservable quality term $(p_{t-1} - \pi_{t-1})$, a_3 above, should equal $\phi(1 - a_2)$ where a_2 is the coefficient on the anchoring regressor above.

Finally, if we assume that the auctioneer's estimates are unbiased predictors of price and let $\Phi(\phi, \tau)$ be represented by Φ, then our final result is equation 1 in the text:

$$p_{it} = a_0 + a_1\pi_{it} + a_2(p_{it-1} - \pi_{it}) + a_3(p_{it-1} - \pi_{it-1}) + a_4\Phi + \eta_{it}$$

Below we explain how this equation can be estimated using non-linear least squares. Consistency and asymptotic normality (and accounting for measurement error) of the estimator follows from Hsiao (1989).

Non-linear Least Squares Estimation

The Grid Search

We start by calculating $\Phi(\phi, \tau)$ for given values of ϕ and τ. The value of $\Phi(\phi, \tau)$ is the expected value of U^+, which is the normal distribution U truncated at zero: a value that depends on both ϕ and τ. The values of the remaining parameters that minimize the sum of squared errors—$SSE(\phi, \tau)$—are determined by OLS. This procedure is repeated over a grid of different values of ϕ and τ to find the overall minimum SSE. The grid was set to have rectangular tiles of 0.05 for divisions of τ and 0.005 for ϕ with values of ϕ ranging between 0 and 1 and the values of $\ln(\tau)$ range between -10 and 2. The restriction on the coefficient for unobservable quality is imposed during the grid search.

The surface that results does not reveal an obvious minimum value for the value of τ, but it provides a solid estimation for ϕ. For this value of ϕ, the SSE is almost entirely flat, but decreasing slowly as τ increases, with one bigger drop when τ is approximately equal to negative one. In addition to resulting in a

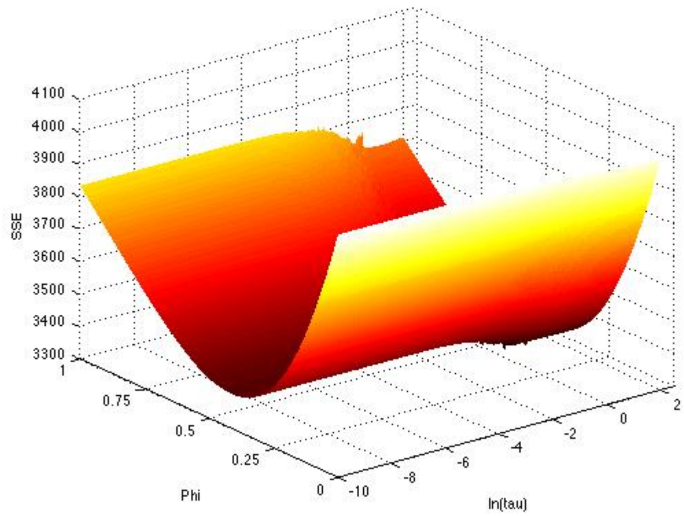


Figure 3: Grid search using the sale price as the dependent variable.

higher SSE, the coefficient estimates for very small values of τ result in all zero estimates for the value of the expected loss term, which does not seem reasonable so we forgo very small values of τ. High values of τ result in coefficients for the loss aversion term that do not make economic sense—although those coefficient values are higher and more significant than the values reported—so we forgo very high values of τ. Since the SSE is slightly decreasing as τ increases, we choose a reasonable value of τ— on the high end of our range for the grid search, but excluding the highest values—and report results for these values of ϕ and τ. For the range of values of τ that produce reasonable results, the point estimates for the coefficients do not vary significantly, so the results are robust to values of τ over a fairly large range.

A hill climbing algorithm could have been used to further refine the value of ϕ once the grid search determined there weren't multiple minima. But this is unnecessary since we re-ran the grid search using a finer grid for ϕ and found

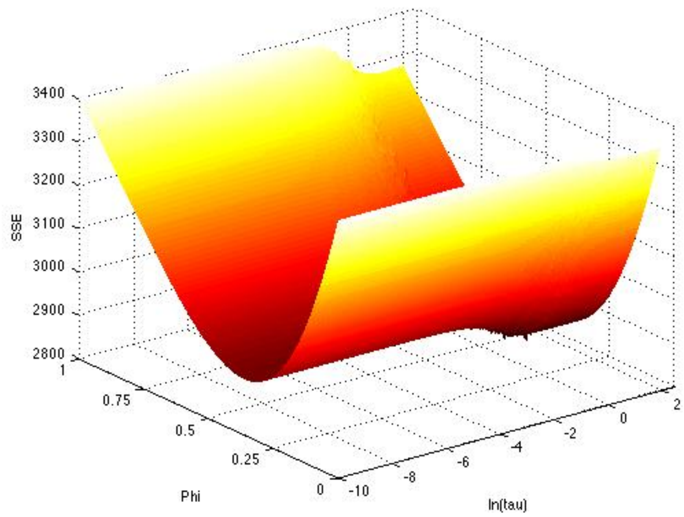


Figure 4: Grid search using the low estimate as the dependent variable.

no significant changes in the results.

Inference

Loss Aversion Regressor

We are interested in testing whether the β value on the loss aversion regressor is significantly different from zero. This is complicated by the fact that when this coefficient is zero, $z(\phi, \tau)$ does not enter the regression and ϕ and τ are not identified. Standard asymptotic theory therefore does not apply, so we use the bootstrap to estimate a standard error or a confidence interval for the coefficient.

In order for a bootstrap to be reasonably accurate, the data generating process used for drawing bootstrap samples should be as close as possible to the true data generating process. Because the errors are heteroskedastic of unknown form, there are two options available: the pairs bootstrap and the wild bootstrap. The pairs bootstrap works by resampling observations with replacement, but it is only generally applicable when the observations are independent, which is not the case in our data. The wild bootstrap works by sampling N numbers from a distribution with mean 0 and variance 1 and then transforming the residuals from the original regression by multiplying them by the random sampling: effectively randomly changing the signs on the residuals. Because It has been shown by Davidson and Flachaire(2008) and Flachaire (2005) that it is the preferred distribution to sample from in almost all practical situations, we sample from the Rademacher distribution:

$$\epsilon_t = \begin{cases} 1 & \text{with probability } 1/2 \\ -1 & \text{with probability } 1/2. \end{cases}$$

The regression is then run by subtracting the transformed residuals from the dependent variable and rerunning the grid search and then the robust regression. This procedure is repeated 2000 times and results in a distribution of 2000 coefficients on the loss aversion term. If that distribution is normal, we report its standard deviation as the standard error. Otherwise we report confidence intervals based on the percentiles of the distribution. The standard errors for the loss aversion regressor, the unobservable regressor, and the nuisance parameters are calculated via the bootstrap.

Unobservable Quality Regressor

Because of the restriction on the coefficient of the unobservable quality regressor (that $a_3 = \phi(1 - a_2)$), it is not identified in the regression. The value of the coefficient is consequently calculated using the estimated values of a_2 and ϕ, but the standard error or confidence interval must be calculated using a bootstrap in the same way as described above for the loss aversion coefficient.

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